Chapter 7 7.1-7.2

1. $\sin^{-1}\frac{1}{2}$

Find the angle θ, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, whose sine equals $\frac{1}{2}$.

$$\sin\theta = \frac{1}{2}, \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$
$$\theta = \frac{\pi}{6}$$

Thus, $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$.

2. $\tan^{-1} 1$

Find the angle θ, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, whose tangent equals 1.

$$\tan\theta = 1, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$
$$\theta = \frac{\pi}{4}$$

Thus, $\tan^{-1}(1) = \frac{\pi}{4}$.

3. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Find the angle θ, $0 \le \theta \le \pi$, whose cosine equals $\frac{\sqrt{3}}{2}$.

$$\cos\theta = \frac{\sqrt{3}}{2}, \quad 0 \le \theta \le \pi$$
$$\theta = \frac{\pi}{6}$$

Thus, $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$.

4. $\cot^{-1}\left(-\sqrt{3}\right)$

$$\cot\theta = -\sqrt{3}, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$
$$\theta = -\frac{\pi}{6}$$

Thus, $\cot^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{6}$.

5. $\sec^{-1}(-1)$

Find the angle θ, $0 \le \theta \le \pi$, whose secant equals -1.

$$\sec\theta = (-1), \quad 0 \le \theta \le \pi$$
$$\theta = \pi$$

Thus, $\sec^{-1}(-1) = \pi$.

6. $\csc^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$

Find the angle θ, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ whose cosecant equals $-\frac{2\sqrt{3}}{3}$.

$$\csc\theta = \left(-\frac{2\sqrt{3}}{3}\right), \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$
$$\theta = -\frac{\pi}{3}$$

7. $\sin^{-1}\left(\sin\left(-\frac{\pi}{8}\right)\right)$ follows the form of the equation $f^{-1}\left(f(x)\right) = \sin^{-1}\left(\sin(x)\right) = x$. Since $-\frac{\pi}{8}$ is in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, we can apply the equation directly and get

$$\sin^{-1}\left(\sin\left(-\frac{\pi}{8}\right)\right) = -\frac{\pi}{8}.$$

8. $\cos\left(\cos^{-1}(1.2)\right)$ follows the form of the equation $f\left(f^{-1}(x)\right) = \cos\left(\cos^{-1}(x)\right) = x$.

Since 1.2 is not in the interval $[-1,1]$, we can apply the equation is not defined

9. $\sin\left(\sin^{-1}(1)\right) = 1$

10. $\tan^{-1}\left(\tan\frac{7\pi}{4}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$

11. $\cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

12. $\tan\left[\cos^{-1}\left(-\frac{1}{2}\right)\right]$

Find the angle θ, $0 \le \theta \le \pi$, whose cosine equals $-\frac{1}{2}$.

$$\cos\theta = -\frac{1}{2}, \quad 0 \le \theta \le \pi$$
$$\theta = \frac{2\pi}{3}$$

So, $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

Thus, $\tan\left[\cos^{-1}\left(-\frac{1}{2}\right)\right] = \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$.

13. $\csc\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)$

Find the angle θ, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, whose sine equals $\frac{\sqrt{3}}{2}$.

$$\sin\theta = \frac{\sqrt{3}}{2}, \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$
$$\theta = \frac{\pi}{3}$$

So, $\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$.

Thus, $\csc\left(\sin^{-1}\frac{\sqrt{3}}{2}\right) = \csc\left(\frac{\pi}{3}\right) = \frac{2\sqrt{3}}{3}$.

14. $\cos\left(\csc^{-1}\frac{5}{3}\right)$

Since $\csc\theta = \frac{5}{3}$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, let $r = 5$ and $y = 3$.

Solve for x: $x^2 + 9 = 25$
$$x^2 = 16$$
$$x = \pm 4$$

Since θ is in quadrant I, $x = 4$.

Thus, $\cos\left(\csc^{-1}\frac{5}{3}\right) = \cos\theta = \frac{x}{r} = \frac{4}{5}$.

15. $\tan\left[\cos^{-1}\left(-\frac{3}{5}\right)\right]$

Since $\cos\theta = -\frac{3}{5}$, $0 \le \theta \le \pi$, let $x = -3$ and $r = 5$. Solve for y: $9 + y^2 = 25$
$$y^2 = 16$$
$$y = \pm 4$$

Since θ is in quadrant II, $y = 4$.

Thus, $\tan\left[\cos^{-1}\left(-\frac{3}{5}\right)\right] = \tan\theta = \frac{y}{x} = \frac{4}{-3} = -\frac{4}{3}$

16. $f(x) = 2\sin(-x+1)$
$$y = 2\sin(-x+1)$$
$$x = 2\sin(-y+1)$$
$$\frac{x}{2} = \sin(-y+1)$$
$$-y+1 = \sin^{-1}\left(\frac{x}{2}\right)$$
$$-y = \sin^{-1}\left(\frac{x}{2}\right) - 1$$
$$y = 1 - \sin^{-1}\left(\frac{x}{2}\right)$$

The domain of $f(x)$ is the set of all real numbers, or $(-\infty, \infty)$ in interval notation. To find the domain of $f^{-1}(x)$ we note that the argument of the inverse sine function is $\frac{x}{2}$ and that it must lie in the interval $[-1, 1]$. That is,
$$-1 \le \frac{x}{2} \le 1$$
$$-2 \le x \le 2$$

The domain of $f^{-1}(x)$ is $\{x \mid -2 \le x \le 2\}$, or $[-2, 2]$ in interval notation.

Recall that the domain of a function is the range of its inverse and the domain of the inverse is the range of the function. Therefore, the range of $f(x)$ is $[-2, 2]$ and the range of $f^{-1}(x)$ is $(-\infty, \infty)$.